'A community of Learners, Believers and Friends'

Numeracy at MSJ

Supporting numeracy across the curriculum

Introduction

What is the purpose of this booklet?

This booklet has been produced to give guidance to pupils and parents on how certain common Numeracy topics are taught during maths lessons at Mount St Joseph. Staff from all departments have access to a copy of the booklet. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

Read through the booklet one section at a time and then try the questions that are set at the end of most sections, checking your answers with those given at the end of the booklet. You can also talk to your child as you go through, asking them questions about the various topics. For example, asking them to describe a parallelogram, or what a negative number multiplied by another negative number gives.

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. Simply look up the relevant page for a step by step guide and useful examples.

This booklet includes skills not only useful in their maths lessons, but also in other subjects across the curriculum and in general outside of school.

For help with maths topics not found in this booklet, pupils should refer to their class work or ask their teacher for help.

Why is their more than one method shown?

In some cases the method used will be dependent on the level of difficulty of the question, whether or not a calculator is permitted or simply which method the pupil themselves prefers.

Table of Contents

1. Mental methods (+ - x ÷)

Addition

Example 54 + 27

Method 1 – Add the tens, then the units, then add together

50 + 20 = 70   4 + 7 = 11   70 + 11 = 81

Method 2 – Split the number to be added into tens and units and add separately.

54 + 20 = 74   74 + 7 = 81

Method 3 – Round up to the next 10, then subtract.

54 + 30 = 84 but 30 is 3 too many therefore subtract 3

84 – 3 = 81

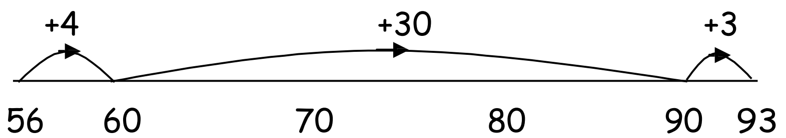
Subtraction

Example 93 - 56

Method 1 – Count on

Count on from 56 until you reach 93.

This can be done in several ways e.g.

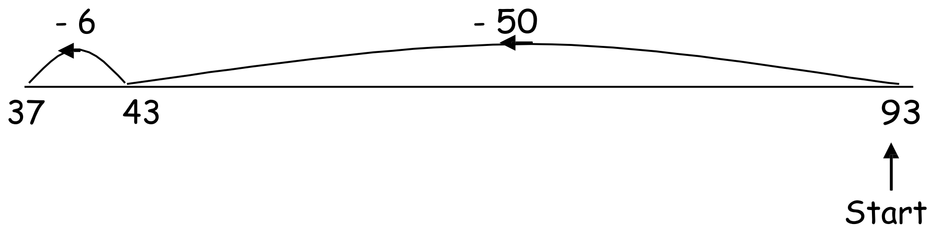


Answer = 37

Method 2 – Break up the number being subtracted

e.g. subtract 50 then subtract 6.

93 - 50 = 43
43 - 6 = 37



Multiplication

It is essential that pupils know all of the times tables from 1x1 up to 10x10. These are shown below:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

7 x 9 = 63

Example 39 x 6

Method 1 – Multiply by the tens then by the units

30 x 6 = 180

9 x 6 = 54

180 + 54 = 234

Method 2 – Multiply 40 x 6 then subtract 1 x 6

40 x 6 = 240

40 is 1 too many so
Subtract 1 x 6 =

240 – 6 = 234

For you to try

1) 56 + 23	2) 69 + 16	3) 436 + 78
4) 45 - 24	5) 84 - 68	6) 537 - 84
7) 23 x 6	8) 59 x 8	9) 7 x 68

2. Written methods (+ - x ÷)

Addition

Example 534 + 2678

Place the digits in the correct "place value" columns with the numbers under each other.
Begin adding in the units column.

Show any carrying in the next column.

	Th	H	T	U
	1	1	1	
		5	3	4
+	2	6	7	8
	3	**2**	**1**	**2**

Subtraction

Example: 7689 - 749

Place the digits in the correct "place value" columns with the numbers under each other.

Begin subtracting in the units column.

You can't subtract 9 from 6 so move 1 ten from the 8 tens to the 6 units to make 16 units.

Note that the same has happened with the hundreds.

	Th	H	T	U
	6 ~~7~~	^{1}6	7 ~~8~~	^{1}6
-		7	4	9
	6	**9**	**3**	**7**

For you to try

1) 556 + 69
2) 678 + 37
3) 856 + 376
4) 8072 + 548
5) 7604 + 269
6) 4576 + 643
7) 556 - 48
8) 856 - 673
9) 1234 - 769
10) 4530 - 667
11) 2378 - 1605
12) 7931 - 3347

Addition of decimals

Example 53.4 + 26.78

Place the digits in the correct "place value" columns with the numbers under each other.
Make sure the decimal points are lined up vertically.
Begin adding in the furthest column on the right.

	T	U	.	1/10	1/100
		1			
	5	3	.	4	
+	2	6	.	7	8
	7	9	.	1	8

Subtraction of decimals

Example: 78.9 – 7.49

Place the digits in the correct "place value" columns with the numbers under each other.

Make sure the decimal points are lined up vertically.
Begin subtracting in the furthest column on the right.

Fill in any gaps with zeros.

	T	U	.	1/10	1/100
	7	8	.	8 ~~9~~	1 0
-		7	.	4	9
	7	1	.	4	1

For you to try

1) 69.7 + 36.8
2) 55.7 + 6.38
3) 5.96 + 68.4
4) 78.76 + 6.5
5) 43.7 + 643.2
6) 7.67 + 673.9
7) 34.8 – 15.2
8) 67.9 – 6.45
9) 543.8 – 74.38
10) 56.23 – 16.9
11) 234.1 – 62.4
12) 328 – 81.3

Multiplication

Geography, multipliers of 6 – fertility rate of 6 – showing how quickly the popn will increase

Method 1 – Grid Method

Example 56 x 34

Separate the 56 and 34 into tens and units.

Multiply the columns with the rows and place the answers in the grey boxes.

x	50	6
30	1500	180
4	200	24

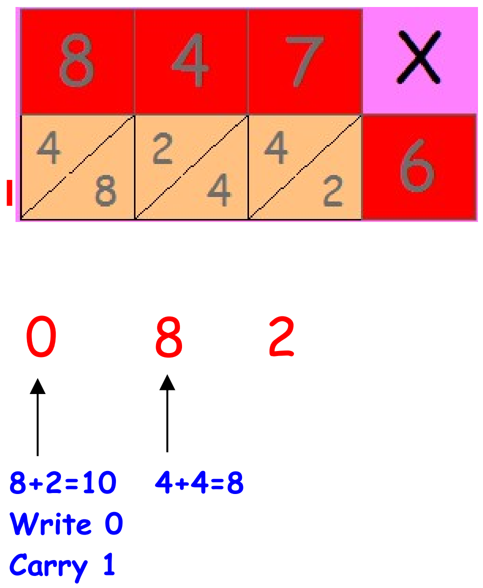
Add the numbers: 1500 + 180 + 200 + 24

= 1904

Method 2 – Napier's Bones

Example 847 x 6

Write 847 across the top and 6 down the side.
Multiply each of the digits 8,4 & 7 by the 6, putting the answers in the orange boxes.
The answer is obtained by adding up from right to left.



For you to try

1) 36 x 62
2) 82 x 47
3) 156 x 5
4) 263 x 7
5) 556 x 62
6) 452 x 81

Division

Example: 980 ÷ 4

Concise method

	2	4	5
4	9	$^{1}8$	$^{2}0$

There are **2** fours in 9 with remainder 1 so the answer starts with **2** and the remainder 1 is placed next to the 8.

There are **4** fours in 18 with remainder 2.

There are **5** fours in 20 with no remainder.

The answer is **245**

Chunking method

We use multiples of 100, 10, 5, 2 and 1 as these are easy to work out.

x	4	Total	
100	400	400	100 x 4 = 400 which is a great deal less than 980.
100	400	800	Another 100 x 4 will make a total of 800.
10	40	840	Another 100 x 4 will give a total of 1200 which is more than 980 so we use 10 x 4 = 40 giving a total of 840.
10	40	880	
10	40	920	10 x 4 = 40 another 3 times gives a total of 960.
10	40	960	
5	20	980	5 x 4 = 20 giving a total of 980 which is what we need.
245			

By adding the "x" column we can see how many 4s there are in 980.

For you to try

1) 558 ÷ 3
2) 624 ÷ 4
3) 266 ÷ 7
4) 1554 ÷ 6
5) 7535 ÷ 5
6) 4203 ÷ 9

3. Number Properties

Even numbers

2, 4, 6, 8, 10, 12, …, etc.

Even numbers are the same as the numbers in the two times table.

A number is even if it ends in a 2, 4, 6, 8 or 0

e.g. 5678 is even as it ends in an 8.

Odd numbers

1, 3, 5, 7, 11, 13, …, etc.

Odd numbers are all the numbers that aren't in the two times table.

A number is odd if it ends in a 1, 3, 5, 7 or 9

e.g. 673 is odd as it ends in a 3.

Square numbers

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$

The first ten square numbers are:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100

For you to try

From the following list which are a) odd b) even c) square numbers?

7, 11, 18, 25, 30, 36, 100, 285, 3498

Multiples

A multiple of a number is that number multiplied by any whole number.

e.g. 14 is a multiple of 7 because 7 x 2 = 14
6 is a factor of 2 because 2 x 3 = 6.

The multiples of a number start with that number and can be thought of as the times table of that number.

e.g. The multiples of 5 are: 5, 10, 15, 20, 25, …, etc.

Note: Multiples of a number go on forever!

Factors

A factor is a number that divides exactly into another number.

e.g. 4 is a factor of 12 because 3 lots of 4 make 12.
6 is a factor of 12 because 2 lots of 6 make 12.

All the factors of 12 are: 1, 2, 3, 4, 6 and 12

Prime numbers

A prime number has exactly **two** factors, 1 and itself.

e.g. The only factors of 17 are 1 and 17. So 17 is a prime number.

The prime numbers between 1 and 20 are:

2, 3, 5, 7, 11, 13, 17, 19

Note: 1 is not a prime number because it only has one factor!

For you to try

From the following list, which are a) multiples of 6 b) factors of 30 c) prime?

3, 5, 9, 12, 15, 19, 24, 30

4. Place Value

Thousands (1000)	Hundreds (100)	Tens (10)	Units (1)	•	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$

10 units = 1 ten
10 tens = 1 hundred
10 hundreds = 1 thousand

10 thousandths = 1 hundredth
10 hundredths = 1 tenth
10 tenths = 1 unit

Geography – 4.6 billion years – they have no concept of how big this number is!

The placement of the digits within the number gives us the value of that digit.

e.g.

4 2 8 4 . 5 6 7

The digit 4 has the value of 4 thousand (4000)

The digit 5 has the value of 5 tenths ($^{5}/_{10}$ or 0.5)

The digit 8 has the value 8 tens (80)

The digit 7 has the value 7 thousandths ($^{7}/_{1000}$ or 0.007)

For you to try

What is the value of the 7 in each of the following numbers?

1) 756
2) 2 578
3) 47 489
4) 4.75
5) 2.07
6) 37 488 234

5. Fractions

Understanding Fractions

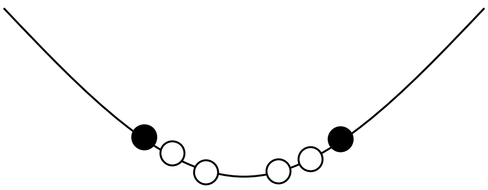
The **numerator** is the number on the top of the fraction

$$\frac{3}{4}$$

The **denominator** is the number on the bottom

Example

A necklace is made from black and white beads.

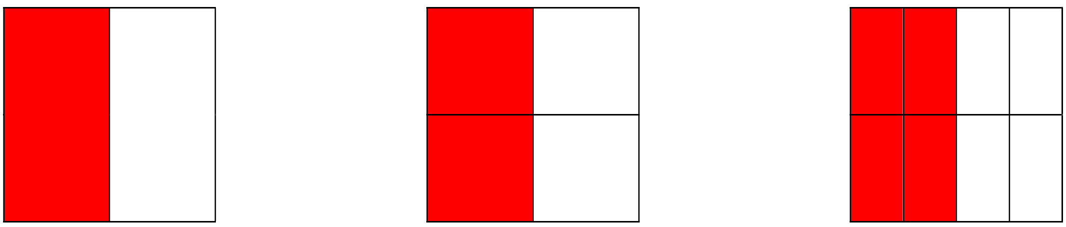


What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent fractions

All the fractions below represent the same proportion. Therefore they are called equivalent fractions.



$^1/_2$ $\quad$ $^2/_4$ $\quad$ $^4/_8$

Below are three rows of equivalent fractions. What do you think would come next?

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \ldots$$

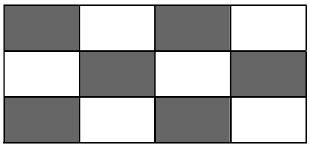
$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \ldots$$

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \ldots$$

You can tell if two fractions are equivalent if the numerator and denominator have both been multiplied by the same amount.

Example

What fraction of the flag is shaded?



6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

× 6

$\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions because 1 × 6 = 6 and 2 × 6 = 12

× 6

Simplifying Fractions

To simplify a fraction you divide the numerator and denominator by the same number.

Example

(a)

$$\frac{20}{25} \overset{\div 5}{=} \frac{4}{5} \quad (\div 5 \text{ top and bottom})$$

(b)

$$\frac{16}{24} \overset{\div 8}{=} \frac{2}{3} \quad (\div 8 \text{ top and bottom})$$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its simplest form.

Example

Simplify $\frac{72}{84}$

$$\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7} \text{ (simplest form)}$$

Fractions of Quantities

To find the fraction of a quantity, divide by the denominator.

To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1

Find $\frac{1}{5}$ of £150

$$\frac{1}{5} \text{ of } £150 = £150 \div 5 = £30$$

Example 2

Find $\frac{3}{4}$ of 48 (To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$)

$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

so $\frac{3}{4}$ of 48 = 3 x 12 = 36

For you to try

1) Write each of the following fractions in their simplest form:

a) $\frac{10}{16}$ b) $\frac{15}{20}$ c) $\frac{8}{12}$

d) $\frac{20}{80}$ e) $\frac{7}{21}$ f) $\frac{24}{40}$

2) Calculate each of the following:

a) $\frac{1}{4}$ of 24 b) $\frac{1}{3}$ of 30 c) $\frac{1}{5}$ of 45

d) $\frac{3}{4}$ of 20 e) $\frac{2}{5}$ of 40 f) $\frac{7}{9}$ of 72

6. Percentages

%

The symbol **%** Means **Out of 100**

63%	Means	$^{63}/_{100}$	(63 out of 100)
100%	Means	$^{100}/_{100}$	(or 1 whole one)
120%	Means	$^{120}/_{100}$	(Percentages can be more than 100%)

Percentages of Amounts

Non- Calculator Methods

Method 1 Using Equivalent Fractions:

$50\% = \frac{1}{2}$ $25\% = \frac{1}{4}$ $10\% = \frac{1}{10}$

Example

a) Find 50% of 2000kg

$$50\% \text{ of } 2000\text{kg} = \frac{1}{2} \text{ of } 2000\text{kg} = 2000\text{kg} \div 2 = 1000\text{kg}$$

b) Find 25% of £640

$$25\% \text{ of } £640 = \frac{1}{4} \text{ of } £640 = £640 \div 4 = £160$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example

Find 9% of 200g

$$1\% \text{ of } 200g = \frac{1}{100} \text{ of } 200g = 200g \div 100 = 2g$$

$$\text{so } 9\% \text{ of } 200g = 9 \times 2g = 18g$$

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example

Find 70% of £35

$$10\% \text{ of } £35 = \frac{1}{10} \text{ of } £35 = £35 \div 10 = £3.50$$

$$\text{so } 70\% \text{ of } £35 = 7 \times £3.50 = £24.50$$

For you to try (without a calculator)

1) 50% of 200
2) 25% of 80
3) 10% of 40
4) 20% of 60
5) 30% of 500
6) 70% of 90
7) 3% of 600
8) 15% of 360
9) 67% of 300

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a fraction, then multiply.

Example

a) Find 23% of £15000

$23\% = \frac{23}{100}$ so 23% of £15000 = 23 ÷100 x £15000 = £3450

A fraction can be typed into a calculator as Numerator ÷ Denominator

b) Find 68% of £400

$68\% = \frac{68}{100}$ so 68% of £400 = 68 ÷100 x £400 = £272

Note: We do not use the % button on a calculator during maths lessons!

For you to try (with a calculator)

1) 24% of 50
2) 79% of 400
3) 18% of 2000
4) 17.5% of 40
5) 47% of 4600
6) 135% of 20

7. Fraction, Decimal & Percentage Equivalence

Some fractions and percentages are used very frequently. It is useful to be able to express these as either a fraction, decimal or percentage.

Fraction	Decimal	Percentage
1	1	100%
$\frac{1}{2}$	0·5	50%
$\frac{1}{3}$	0·33.....	33%
$\frac{1}{4}$	0·25	25%
$\frac{3}{4}$	0·75	75%
$\frac{1}{10}$	0·1	10%
$\frac{2}{10}$ (= $\frac{1}{5}$)	0·2	20%
$\frac{3}{10}$	0·3	30%

For you to try

1) Change into decimals: a) 40% b) 85% c) $\frac{7}{10}$

2) Change into percentages: a) 0.8 b) $\frac{5}{10}$ c) $\frac{4}{5}$

3) Change into fractions: a) 90% b) 0.6 c) 0.4

8. Ratio & Proportion

Writing a Ratio

Ratio is used to make a comparison between two things.

Example

☺ ☺ ☺ ☹ ☺ ☺ ☺ ☹ ☺ ☺ ☺ ☹

In this pattern we can see that there are 3 happy faces **to** every sad face.
We use the symbol **:** to represent **to** in the above statement, therefore we write the ratio like this:

The ratio of happy faces to sad faces is 3 : 1

The ratio of sad faces to happy faces is 1 : 3

Note: The order of the numbers is important.

Ratio is used in a number of situations including

- In a cooking recipe
- When mixing concrete or paint
- In the scale on maps or in models
  e.g. if a scale of **1 : 100 000** is used on a map, it means that **1 cm** on the map represents **100 000 cm** in reality.

Simplifying Ratios

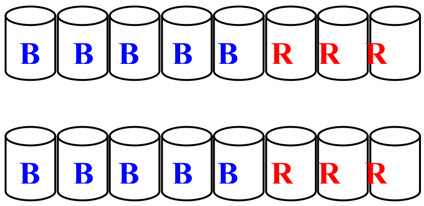
Ratios can be simplified in much the same way as fractions, by dividing each part of the ratio by the same number

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red.

The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



We have simplified the ratio 10 : 6 by dividing both numbers by two to get 5 : 3

Example 2

Simplify each ratio:

(a) 4:6

(b) 24:36

(c) 6:3:12

(a) 4:6 (Divide by 2)
= 2:3

(b) 24:36 (Divide by 12)
= 2:3

(c) 6:3:12 (Divide by 3)
= 2:1:4

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement.

Write the ratio of sand : cement in its simplest form

The ratio of Sand to Cement = 20 : 4

Which can be simplified (by dividing by 4) to 5 : 1

Proportion

Two quantities are said to be in direct proportion if when one doubles the other doubles. We can use proportion to solve problems.

Example 1

A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
x3	x3
90	**4500**

The factory would produce 4500 cars in 90 days.

Example 2

5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

Tickets	Cost	
5	£27.50	
1	£5.50	(27.50 ÷ 5)
8	£44.00	(5.50 x 8)

The cost of 8 tickets is £44

For you to try

1) Simplify the following ratios as much as possible:

a) 15 : 12 b) 20 :30 c) 36 : 27 d) 28 : 35 : 14

2) If 3 pens cost 75p, how much would 7 identical pens cost?

3) In a class of 30 pupils there are 18 boys. Write as a ratio in its simplest form the number of boys to the number of girls.

9. Negative Numbers

The negative sign (-) tells us the number is below zero e.g. **-4**. The number line is useful when working with negative numbers. Below is a part of the number line.

Negative direction ← → Positive direction

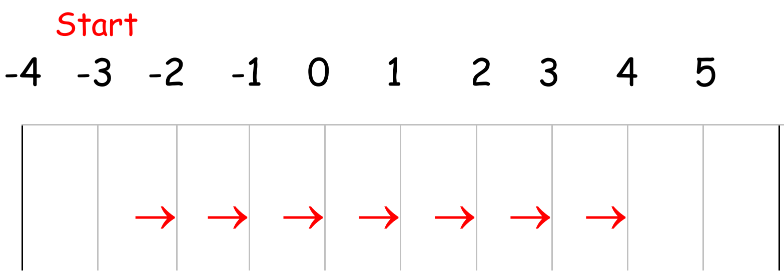
-9	-8	-7	-6	-5	**-4**	-3	-2	-1	0	1	2	3	4	5	6

The numbers on the right are greater than the numbers on the left e.g. 5 is greater than 2 and 2 is greater than -3.

Note that -3 is greater than -8.

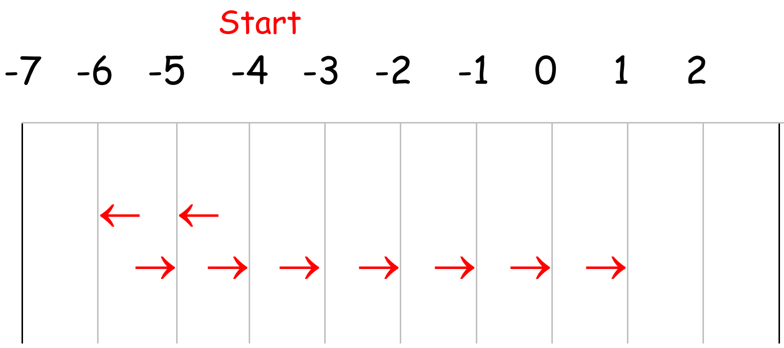
Adding and subtracting with directed numbers

Example: - 3 + 7



Start at -3.
Move 7 in the positive direction
Answer : 4

Example 2: -4 - 2 + 7



Start at -4.
Move 2 in the negative direction
Move 7 in the positive direction
Answer : 1

Multiplying and dividing negative numbers

We multiply and divide negative numbers in the usual way whilst remembering these very important rules:

Two signs the same, a positive answer.
Two different signs, a negative answer.

×	+	-
+	+	-
-	-	+

÷	+	-
+	+	-
-	-	+

Note: If there is no sign before the number, it is positive.

Examples:

5	x	-7	=	-35	(different signs give a negative answer)
-4	x	-8	=	32	(two signs the same give a positive answer)
48	÷	-6	=	-8	(different signs give a negative answer)
-120	÷	-10	=	12	(two signs the same give a positive answer)

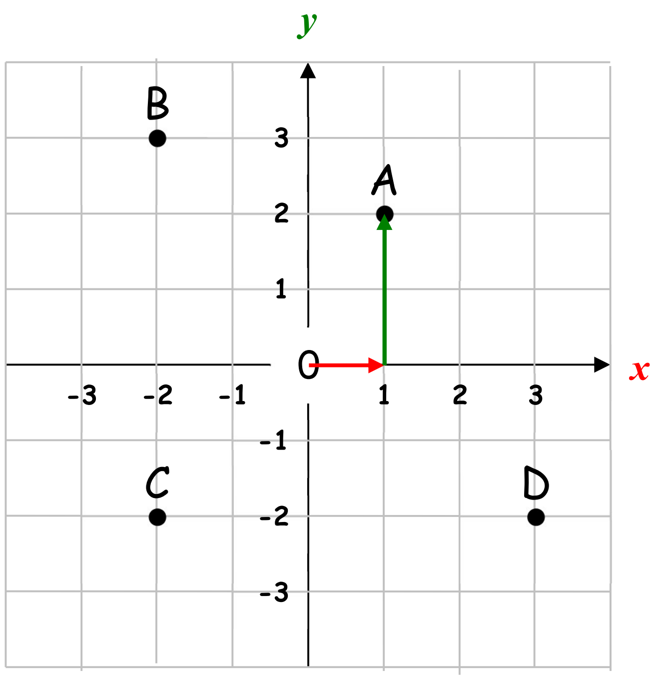
For you to try

1) -8 + 12
2) -5 - 4
3) 12 - 20
4) -15 + 9
5) -4 + 9 - 13
6) -5 × 6
7) -4 × -8
8) -30 ÷ 6
9) -63 ÷ -7

10. Coordinates

Co ordinates of maps (global), and OS maps (grid references)

We use coordinates to describe location. We write a coordinate as two numbers in a bracket separated by a comma. The first number is the x-coordinate (across) and the second number is the y-coordinate (up or down).



Example

The coordinates of the points are:

A=(1,2) (1 across, 2 up) **B=(-2,3)** **C=(-2,-2)** **D=(3,-2)**

Note: There is a special name for the point **(0,0)**. It is called **the origin**.

For you to try

Plot each of the following points on the coordinate grid above:

1) E = (3,3) 2) F = (1,-2) 3) G = (-2,1)

4) H = (-3,0) 5) I = (2,3) 6) J = (-1,-3)

11. Inequalities

We us the = sign to show that two sums are **equal**. If one sum is greater than or less than the other we use inequalities:

$<$ less than	$>$ greater than
$\leq$ less than or equal to	$\geq$ greater than or equal to

Examples :

$5 < 8$ **$43 > 6$** **$-3 > -10$**

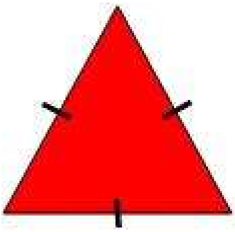
For you to try

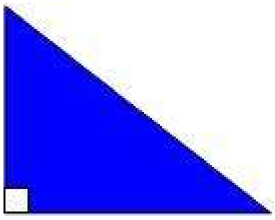
Put the correct symbol, either $<$ or $>$ in between each of the following pairs of numbers:

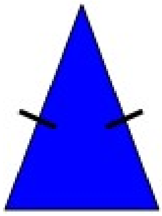
1) 3 5	2) 65 28	3) -5 -12
4) 8 -4	5) -7 -10	6) -4.5 -3

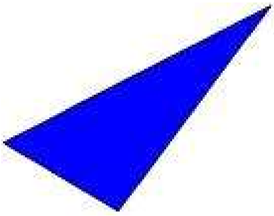
12. Names of two dimensional shapes

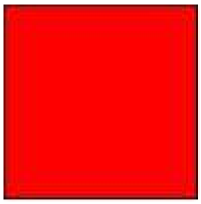
A polygon is a closed shape made up of straight lines.
A **regular polygon** has all of its sides equal in length and all of its angles equal in size.

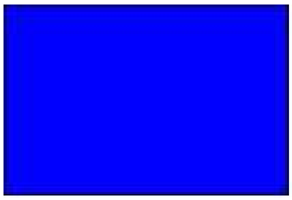

Equilateral triangle

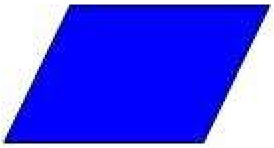

Right angled triangle

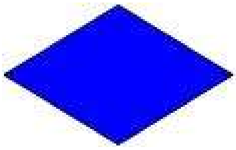

Isosceles triangle

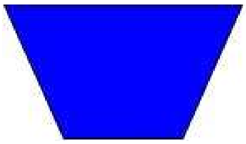

Scalene triangle

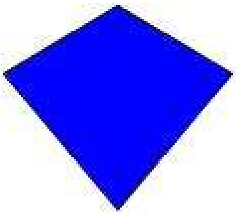

Square

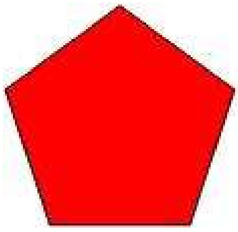

Rectangle

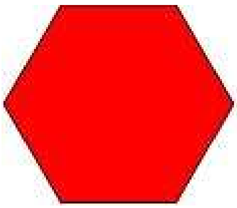

Parallelogram
Opposite sides parallel and equal.

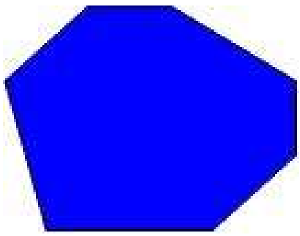

Rhombus
Opposite sides parallel, all sides equal.

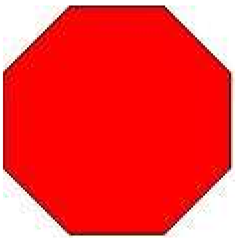

Trapezium
One pair of opposite sides parallel.

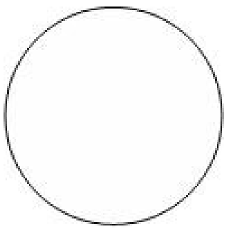

Kite


Pentagon


Hexagon


Heptagon

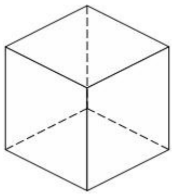
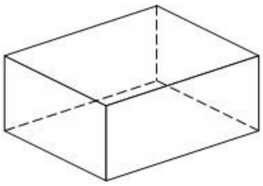
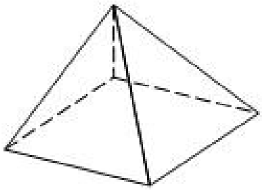
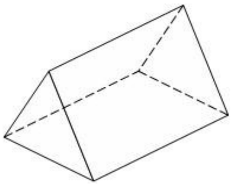

Octagon


Circle

Note: All 2D shapes with 4 sides are known as quadrilaterals

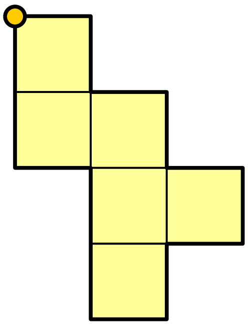
13. 3D shapes

3D means three dimensions – 3D shapes have length, width and height.

Shape	Name	Faces	Edges	Vertices (corners)
	Cube	6	12	8
	Cuboid	6	12	8
	Square based pyramid	5	8	5
	Triangular prism	5	9	6

14. Perimeter

Perimeter is the distance around the outside of a shape. We measure the perimeter in millimetres (mm), centimetres (cm), metres (m), etc.



This shape has been drawn on a 1cm grid. Starting on the orange circle and moving in a clockwise direction, the distance travelled is . . .

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 1 + 2 = 14cm

Perimeter = 14cm

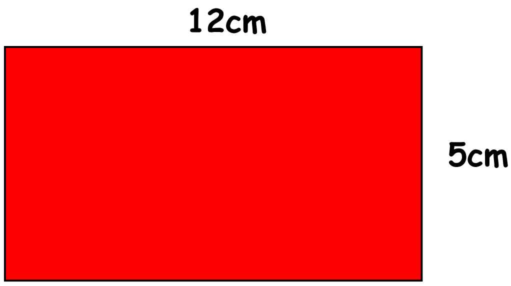
If you know the length of the sides of a shape then to find the perimeter you simply add the lengths together.

Example

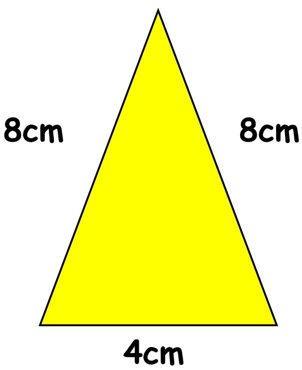
In the rectangle on the right the perimeter =

12 + 12 + 5 + 5 = 34cm

Note: we added the 12cm twice as the bottom edge is equal in size to the top and similarly we added the 5cm twice as the left and right edges are equal.



Example 2



In the triangle on the left the perimeter =

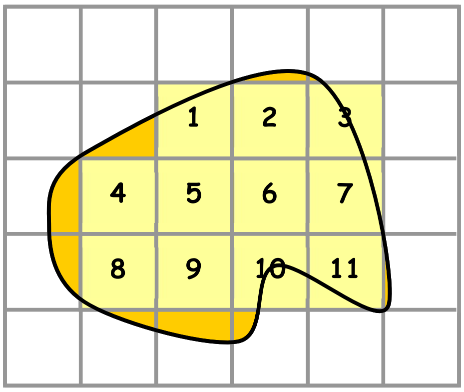
8 + 8 + 4 = 20cm

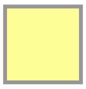
15. Area of 2D Shapes

The area of a shape is how much surface it covers. We measure area in square units e.g. centimetres squared (cm^2) or metres squared (m^2).

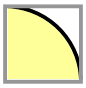
Areas of irregular shapes

Given an irregular shape, we estimate its area through drawing a grid and counting the squares that cover the shape.

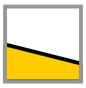




Whole square – count as one.



Half a square or more – count as one.



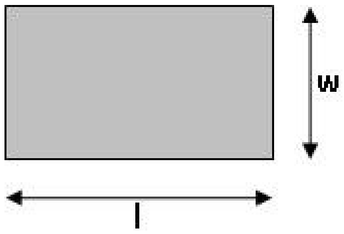
Less than half a square - ignore.

Area = $11cm^2$.

Note: This answer is approximate and not the exact answer.

Area formulae

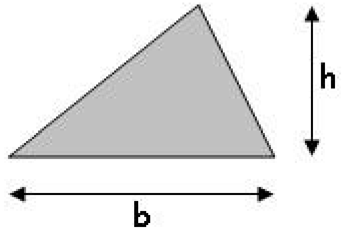
Rectangle



Multiply the length with the width.

Area = $l \times w$

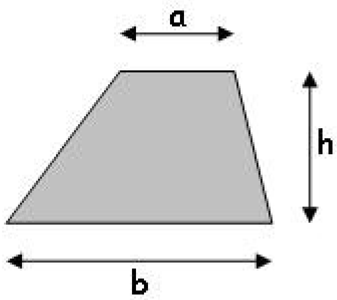
Triangle



Multiply the base with the height and divide by two.

Area = $\dfrac{b \times h}{2}$

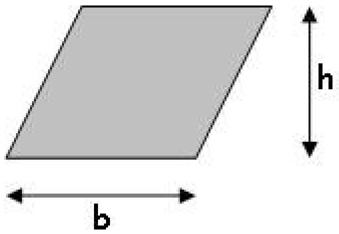
Trapezium



Add the parallel sides, multiply with the height and divide by two.

Area = $\dfrac{(a + b)\,h}{2}$

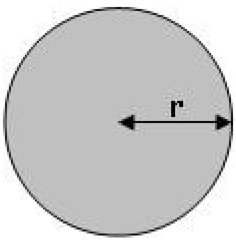
Parallelogram



Multiply the base with the height.

Area = $b \times h$

Circle



Multiply the radius with itself, then multiply with π.

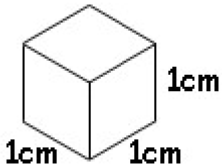
Area = $r \times r \times \pi = \pi r^2$

16. Volume

Volume is the amount of space that an object contains or takes up. The object can be a solid, liquid or gas.

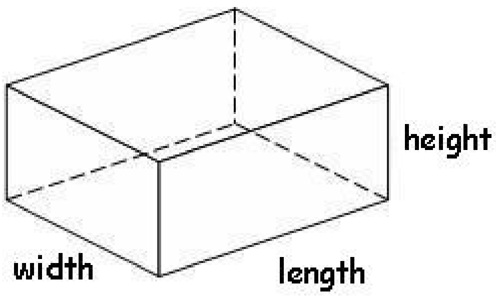
Volume is measured in cubic units e.g. cubic centimetres (cm^3) and cubic metres (m^3).

This cube has a volume of 1 cm^3



Cuboid

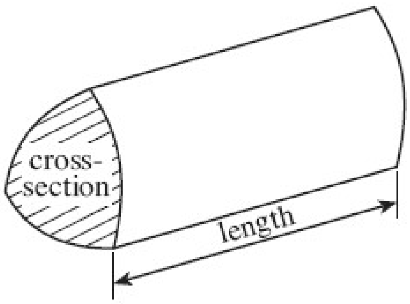
Note that a cuboid has six rectangular faces.



Volume of a cuboid = length x width x height

Prism

A prism is a 3-dimensional object that has the same shape throughout its length
i.e. it has a uniform cross-section.



Volume of a prism = area of cross-section x length

17. Units of Measurement

Metric (new) units of length

Millimetre	Mm	10 mm = 1 cm
Centimetre	Cm	100 cm = 1 m
Metre	M	1 000 m = 1 km
Kilometre	Km	

Imperial (old) units of length

Inch	in or "	12 in = 1 ft
Foot	ft or '	3 ft = 1 yd
Yard	Yd	1 760 yd = 1 mile
Mile		

Metric units of mass

Milligram	Mg	1 000 mg = 1 g
Gram	G	1 000 g = 1 kg
Kilogram	Kg	1 000 kg = 1 t
Metric tonne	T	

Imperial units of mass

Ounce	Oz	16 oz = 1 lb
Pound	Lb	14 lb = 1 st
Stone	St	

Metric units of volume

Millilitre	Ml	1 000 ml = 1 l
Litre	L	

Imperial units of volume

Pint	Pt	8 pt = 1 gal
Gallon	Gal	

Converting between imperial and metric units

Length

1 inch	≈	2.5 cm
1 foot	≈	30 cm
1 mile	≈	1.6 km
5 miles	≈	8 km

Weight/Mass

1 pound	~	454 g
2.2 pounds	~	1 kg

Volume

1 gallon	≈	4.5 litre
1 pint	≈	0.6 litre(568 ml)
$1\frac{3}{4}$ pints	≈	1 litre

For you to try

1) Change into centimetres:	a) 40 mm	b) 230 mm	c) 1.2 m
2) Change into metres:	a) 300 cm	b) 1.5 km	c) 70 cm
3) Change into grams:	a) 2 kg	b) 5 $\frac{1}{2}$ kg	c) 0.3 kg
4) Change into miles:	a) 16 km	b) 80 km	c) 32 km

18. Time

1000	years	=	1 millennium
100	years	=	1 century
10	years	=	1 decade
60	seconds	=	1 minute
60	minutes	=	1 hour
24	hours	=	1 day
7	days	=	1 week
12	months	=	1 year
52	weeks	≈	1 year
365	days	≈	1 year
366	days	≈	1 leap year

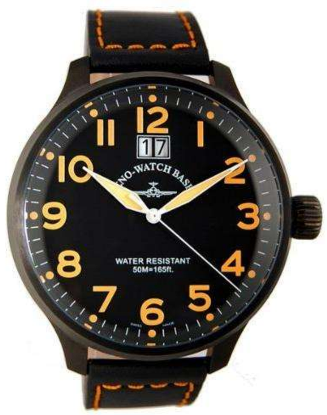
Thirty days hath September
April, June and November,
All the rest have thirty one,
Save February, with twenty eight days clear
And twenty nine each leap year.

The Yearly Cycle

Season	Month	Days
●	January	31
●	February	28
●	March	31
●	April	30
●	May	31
●	June	30
●	July	31
●	August	31
●	September	30
●	October	31
●	November	30
●	December	31

The 24 hour and 12 hour clock

<table>
<tr><th></th><th>24 hour</th><th></th><th>12 hour</th><th></th></tr>
<tr><td>Midnight</td><td>00:00</td><td></td><td>12.00 a.m.</td><td>Midnight</td></tr>
<tr><td rowspan="11">The 24 hour clock always uses 4 digits to show the time.

The 24 hour system does not use a.m. nor p.m.</td><td>01:00</td><td></td><td>1:00 a.m.</td><td rowspan="11">The 12 hour clock shows the time with a.m. before mid-day and p.m. after mid-day.</td></tr>
<tr><td>02:00</td><td></td><td>2:00 a.m.</td></tr>
<tr><td>03:00</td><td></td><td>3:00 a.m.</td></tr>
<tr><td>04:00</td><td></td><td>4.00 a.m.</td></tr>
<tr><td>05:00</td><td></td><td>5:00 a.m.</td></tr>
<tr><td>06:00</td><td></td><td>6:00 a.m.</td></tr>
<tr><td>07:00</td><td></td><td>7:00 a.m.</td></tr>
<tr><td>08:00</td><td></td><td>8:00 a.m.</td></tr>
<tr><td>09:00</td><td></td><td>9:00 a.m.</td></tr>
<tr><td>10:00</td><td></td><td>10:00 a.m.</td></tr>
<tr><td>11:00</td><td></td><td>11:00 a.m.</td></tr>
<tr><td>Mid-day</td><td>12:00</td><td></td><td>12:00 p.m.</td><td>Mid-day</td></tr>
<tr><td rowspan="11"></td><td>13:00</td><td></td><td>1:00 p.m.</td><td rowspan="11"></td></tr>
<tr><td>14:00</td><td></td><td>2:00 p.m.</td></tr>
<tr><td>15:00</td><td></td><td>3:00 p.m.</td></tr>
<tr><td>16:00</td><td></td><td>4:00 p.m.</td></tr>
<tr><td>17:00</td><td></td><td>5:00 p.m.</td></tr>
<tr><td>18:00</td><td></td><td>6:00 p.m.</td></tr>
<tr><td>19:00</td><td></td><td>7:00 p.m.</td></tr>
<tr><td>20:00</td><td></td><td>8:00 p.m.</td></tr>
<tr><td>21:00</td><td></td><td>9.00 p.m.</td></tr>
<tr><td>22:00</td><td></td><td>10.00 p.m.</td></tr>
<tr><td>23:00</td><td></td><td>11:00 p.m.</td></tr>
</table>

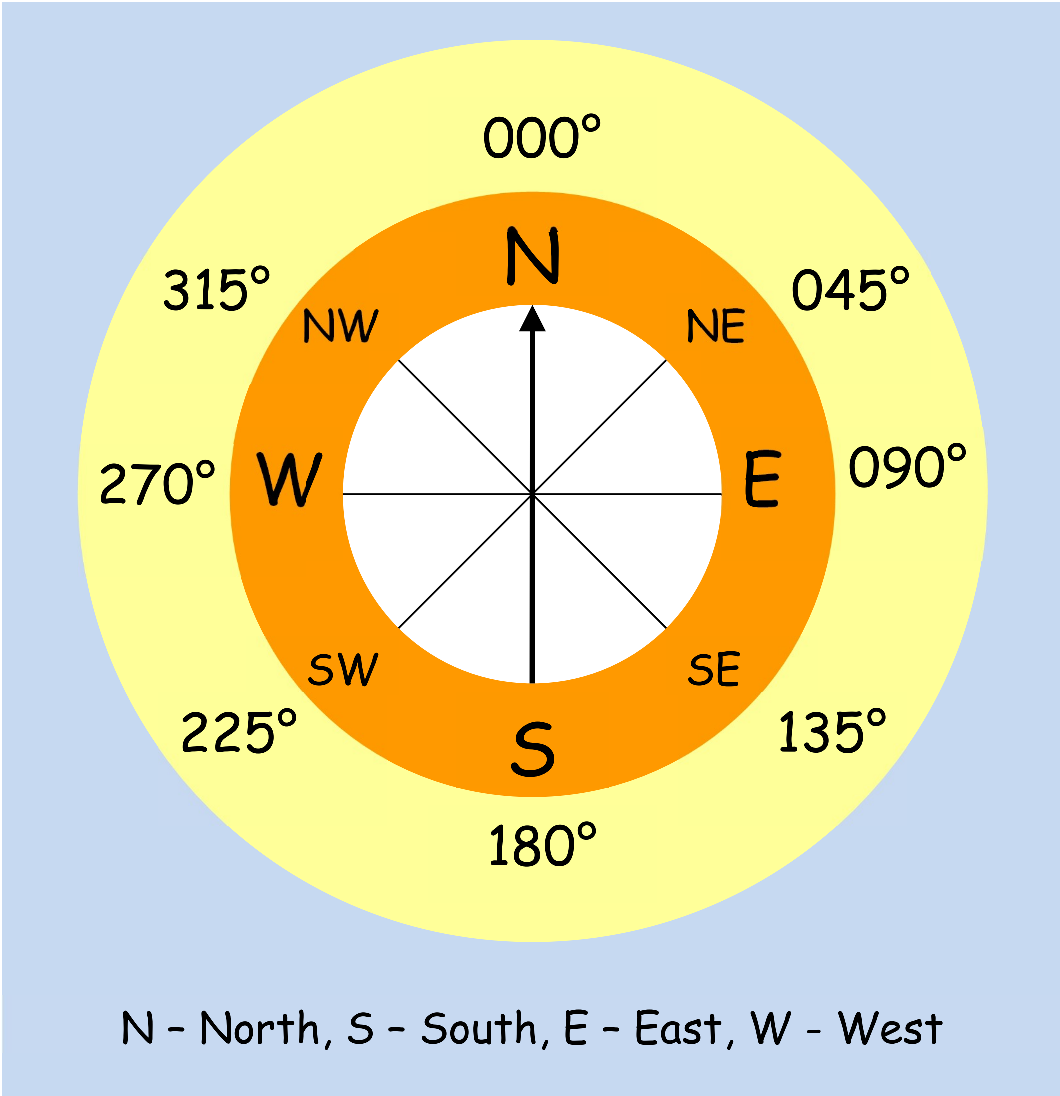
Time vocabulary

02:10	Ten past two in the morning	2:10 a.m.
07:15	Quarter past seven in the morning	7:15 a.m.
15:20	Twenty past three in the afternoon	3:20 p.m.
21:30	Half past nine in the evening	9:30 p.m.
14:40	Twenty to three in the afternoon	2:40 p.m.
21:45	Quarter to ten at night	9:45 p.m.

19. Bearings

A bearing describes direction. A compass is used to find and follow a bearing.

The diagram below shows the main compass points and their bearings.



The bearing is an angle measured clockwise from the North.

Bearings are always written using three figures. e.g. if the angle from the North is 5°, we write 005°.

20. Displaying Data

Collecting and recording

We can record data in a list

e.g. here are the numbers of pets owned by pupils in form 9C:

1 , 2 , 1 , 1 , 2 , 3 , 2 , 1 , 2 , 1 , 1 , 2 , 4 , 2 , 1 , 5 , 2 , 3 , 1 , 1 , 4 , 10 , 3 , 2 , 5 , 1

A frequency table (or tally chart) is more structured and helps with processing the information.

Number of pets	Tally	Frequency
1	~~IIII~~ ~~IIII~~	10
2	~~IIII~~ III	8
3	III	3
4	II	2
5	II	2
6		0
7		0
8		0
9		0
10	I	1

Displaying

In order to communicate information, we use statistical diagrams. Some of the ones we use are:

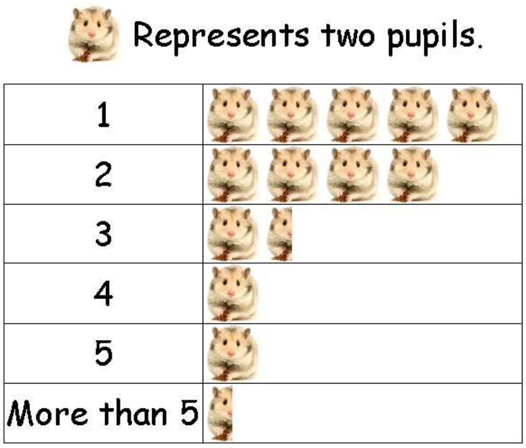
- Pictogram
- Bar Chart
- Pie Chart
- Line Graph
- Conversion Graph
- Scatter diagram

Pictogram

A pictogram uses symbols to represent frequency. We include a key to show the value of each symbol.

Example

The diagram below shows the number of pets owned by pupils in 9C.



We can see that there are 10 pupils that have 1 pet (5 pictures each worth 2).

There are 8 pupils that have 2 pets.

There are 3 pupils that have 3 pets (The $\frac{1}{2}$ picture is worth 1 pupil).

There are 2 pupils that have 4 pets.

There are 2 pupils that have 5 pets.

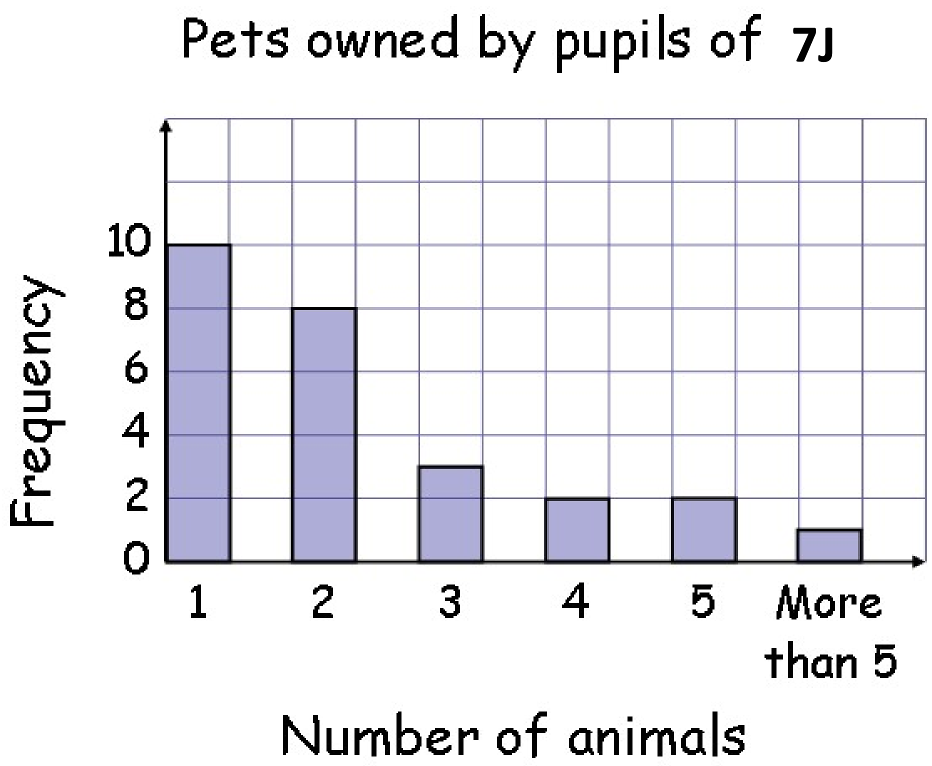
There is 1 pupil that has more than 5 pets.

Bar chart

The height of each bar represents the frequency.

All bars must be the same width and there must be gaps between the bars, also of an equal size.

The scale of the frequency starts from 0 every time and the numbers go next to the lines, not the spaces.



Pie chart

The complete circle represents the total frequency. The angles for each sector are calculated as follows:

Here is the data for the types of pets owned by 9C

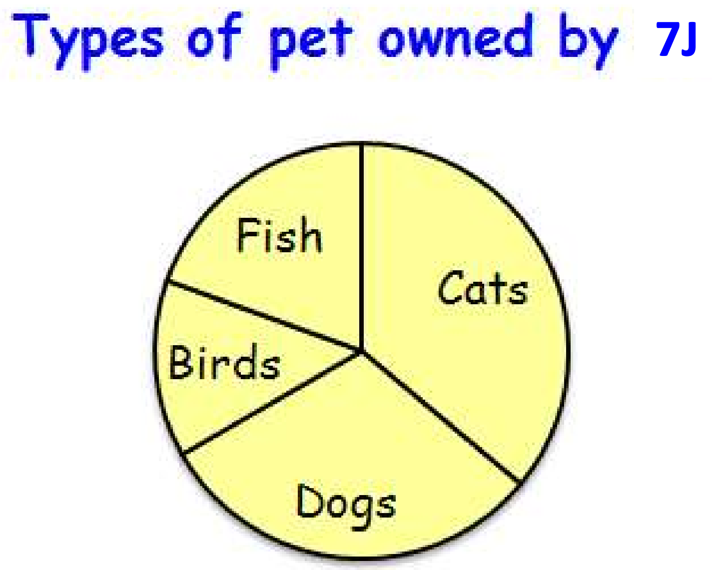
Type of pet	Frequency	Angle of the sector
Cats	13	13 X 10° = 130°
Dogs	11	11 X 10° = 110°
Birds	5	5 X 10° = 50°
Fish	7	7 X 10° = 70°
Total	36	360°

Divide 360° by the total of the frequency:

360° ÷ 36 = 10°

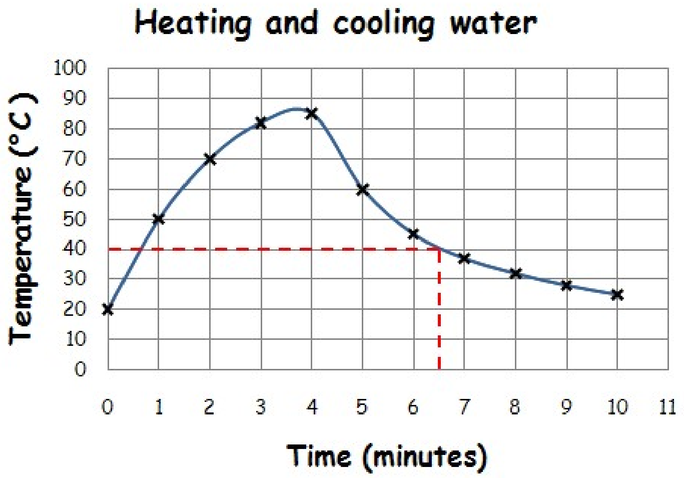
Therefore 10° represents one animal

Remember to check that the angles of the sectors add up to 360°.



Line graph

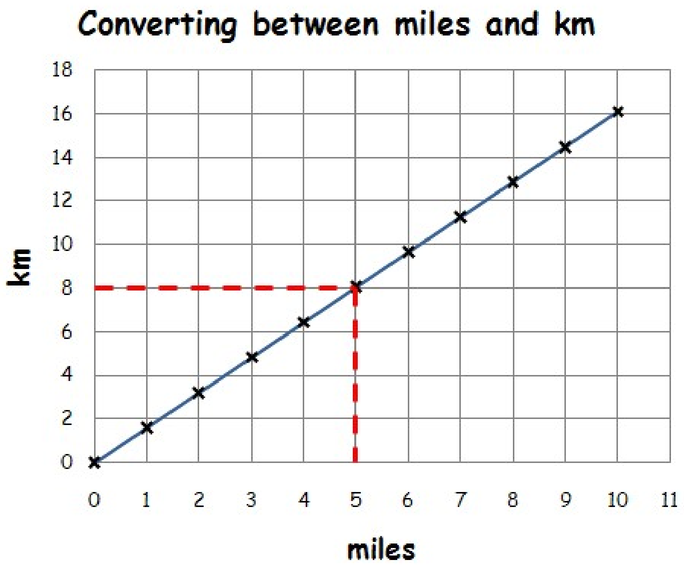
The temperature of water was measured every minute as it was heated and left to cool. A cross shows the temperature of the water at a specific time. Through connecting the crosses with a curve we see the relationship between temperature and time.



The line enables us to estimate the temperature of the water at times other than those plotted e.g. at $6\frac{1}{2}$ minutes the temperature was approximately 40 °C.

Conversion graph

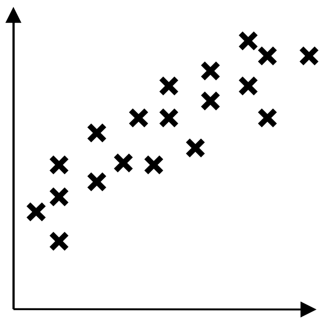
We use a conversion graph for two variables which have a linear relationship. We draw it in the same way as the above graph but the points are connected with a straight line.



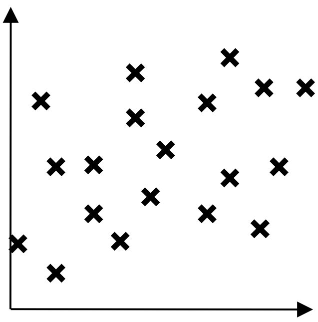
From the graph, we see that 8 km is approximately 5 miles.

Scatter diagram

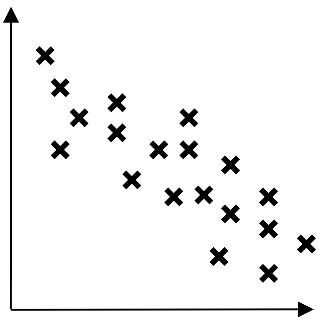
We plot points on the scatter diagram in the same way as for the line graph. We do not join the points but look for a correlation between the two sets of data.



Positive correlation



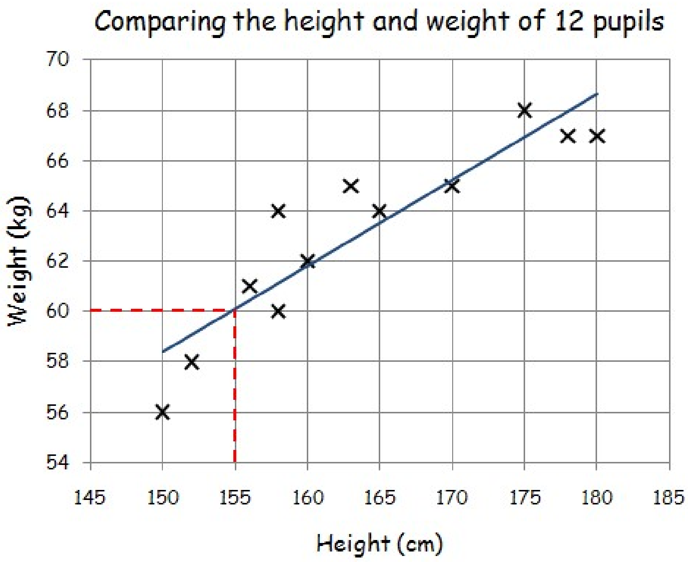
No correlation



Negative correlation

If there is a correlation, we can draw a line of best fit on the diagram and use it to estimate the value of one variable given the other.

The following scatter graph shows a positive correlation between the weights and heights of 12 pupils.



The line of best fit estimates the relationship between the two variables.
Notice that the line follows the trend of the points.
There are approximately the same number of points above and below the line.

We estimate that a pupil 155 cm tall has a weight of 60 kg.

21. Averages & Spread

Averages

The average is a measure of the middle of a set of data. We use the following types of average:

Mean - We add the values in a set of data, and then divide by the number of values in the set.

Median - Place the data in order starting with the smallest then find the number in the middle. This is the median.

If you have two middle numbers then find the number that's halfway between the two.

Mode - This is the value or values that appear most often.

Spread

The spread is a measure of how close together the items of data are. We use the range to measure spread:

Range - The range of a set of data is the difference between the highest and the lowest value.

Example

Find the mean, median, mode and range of the following set of numbers:

4 , 3 , 2 , 0 , 1 , 3 , 1 , 1 , 4 , 5

Mean $\dfrac{4+3+2+0+1+3+1+1+4+5}{10} = 2{\cdot}4$

Median 0 , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , 5 $\dfrac{2+3}{2} = 2{\cdot}5$

Mode 0 , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , 5 = 1

Range 0 , 1 , 1 , 1 , 2 , 3 , 3 , 4 , 4 , 5 5 - 0 = 5

For you to try

Find the mean, median, mode and range of the following set of numbers:

1) 8, 11, 6, 8, 2, 15, 20

2) 6, 7, 8, 10, 3, 12, 15, 8, 6, 5

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 50 – 36 = 14
Division (÷)	Sharing a number into equal parts. 24 ÷ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. 10 > 6
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p46)

Median	Another type of average - the middle number of an ordered set of data (see p46)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p46)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done remembered with the acronym BIDMAS.
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a value of 500.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

Answers

Page 6

1) 77	2) 85	3) 514
4) 21	5) 16	6) 45
7) 138	8) 472	9) 476

Page 7

1) 625	2) 715	3) 1232
4) 8620	5) 7873	6) 5219
7) 508	8) 183	9) 465
10)3863	11) 773	12) 4584

Page 8

1) 106.5	2) 62.08	3) 74.36
4) 85.26	5) 686.9	6) 681.57
7) 19.6	8) 61.45	9) 469.42
10)39.33	11) 171.7	12) 246.7

Page 9

1) 2232	2) 3854	3) 780
4) 1841	5) 34472	6) 36612

Page 10

1) 186	2) 156	3) 38
4) 259	5) 1507	6) 467

Page 11

a) 7, 11, 25, 285
b) 18, 30, 36, 100, 3498
c) 25, 36, 100

Page 12

a) 12, 24, 30
b) 3, 5, 15, 30
c) 3, 5, 19

Page 13

1) 700	2) 70	3) 7000
4) 0.7 or 7/10	5) 0.07 or 7/100	
6) 7 000 000		

Page 17

1) a) 5/8	b) 3/4	c) 2/3
d) 1/4	e) 1/3	f) 3/5
2) a) 6	b) 10	c) 9
d) 15	e) 16	f) 56

Page 19

1) 100	2) 20	3) 4
4) 12	5) 150	6) 63
7) 18	8) 54	9) 201

Page 20

1) 12	2) 316	3) 360
4) 7	5) 2162	6) 27

Page 21

1) a) 0.4 b) 0.85 c) 0.7

2) a) 80% b) 50% c) 80%

3) a) 9/10 b) 6/10 or 3/5

c) 4/10 or 2/5

Page 24

1) a) 5:4 b) 2:3 c) 4:3
d) 4:5:2

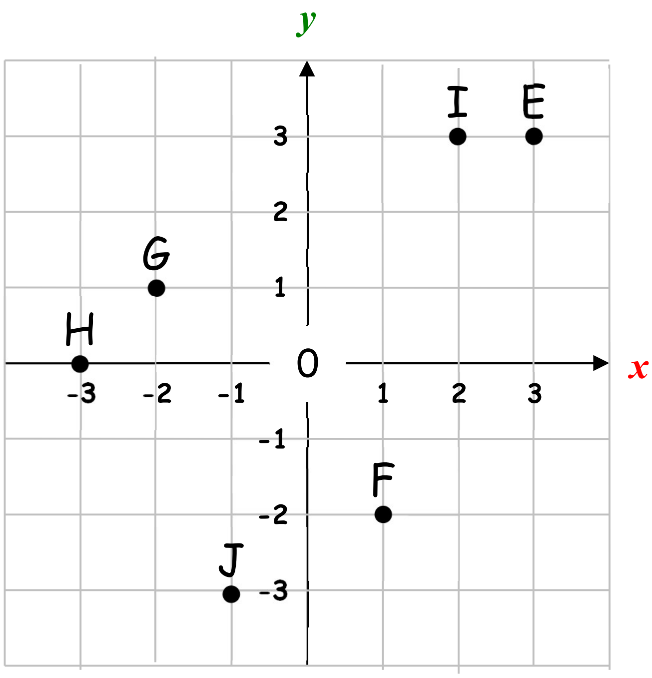
2) £1.75

3) 3:2

Page 26

1) 4	2) -9	3) -8
4) -6	5) -8	6) -30
7) 32	8) -5	9) 9

Page 27



Page 28

1) 3 < 5	2) 65 > 28	3) -5 > -12
4) 8 > -4	5) -7 > -10	6) -4.5 < -3

Page 36

1) a) 4cm	b) 23cm	c) 120cm
2) a) 3m	b) 1500m	c) 0.7m
3) a) 2000g	b) 5500g	c) 300g
4) a) 10miles	b) 50miles	c) 20miles

Page 47

1) Mean = 10, Median = 8, Mode = 8
Range = 18

2) Mean = 8, Median = 7.5, Mode = 6&8
Range = 12

How you can help your child at home

- It is most important that you *talk & listen* to your child about their work in maths. It will help your child if they have to explain to you.
- Share the maths activity with your child and discuss it with them.
- Be positive about maths, even if you don't feel confident about it yourself.
- Remember, you are not expected to teach your child maths, but please share, talk and listen to your child.
- If your child cannot do their homework do let the teacher know by either writing a note in your child's book or telling the teacher.
- A lot of maths can be done using everyday situations and will not need pencil and paper methods.
- Play games and have fun with maths!

Here are some examples of how you can include mathematics at home:

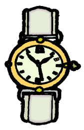
Shopping & Money

£ Looking at prices
£ Calculating change – which coins, different combinations.
£ Counting pocket money.
£ Reading labels on bottles, packets, in order to discuss capacity, weight, shape and colour.

£ Estimating the final bill at the end of shopping while waiting at the checkout.
£ Calculating the cost of the family going to the cinema, swimming baths, etc.

Time

- Looking at the clock – telling the time using analogue and digital clocks.
- Calculating how long a journey will take looking at train/bus/airline timetables.
- Using a TV guide to calculate the length of programmes.
- Looking at the posting times on the post box.



- Discussing events in the day e.g. teatime, bed time, bath time.
- Setting an alarm clock.